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AN ALGORITHM AND ANALYTICAL METHOD FOR CALCULATING THERMALLY INDUCED ABERRATIONS OF MIRRORS

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A general algorithm for the analytical determination of thermooptical aberrations of concave mirrors is suggested. Using as an example mirrors of parabolic profile, analytical relationships are obtained in the geometric optics approximation.

The experience of theoretical and experimental investigations of the thermal aberrations of optical systems points to the necessity of developing analytical computational-theoretical techniques [1-4]. Analytical computational procedures are indispensable for solving inverse problems and for interpreting experimentally observed facts and results of calculations with the use of modern application packages that differ little from experimental data in informative value and generalizations. Only analytical procedures enable one to predict the general trends and, more importantly, to study the threshold conditions for the formation of thermal distortions in those cases where numerical calculation turns out to be unstable and yields poorly reproducible results.

The present paper presents an algorithm for obtaining an analytical solution of a straight-through problem of determining thermal aberrations from initial given conditions of thermal effects. The algorithm incorporates the stages of the solution of a thermal, thermoelastic, and an optical problem. The proposed approach, whose general idea was outlined earlier in [1-4], is realized in the form of an algorithm distinguished by its compact form as compared with alternative schemes of conclusions [1-3]. The algorithm for computing aberrations from displacements has generality for any thermal effects.

However, it is evident that to obtain a particular form of the final analytical solution it is necessary to specify the conditions of thermal effects and the conditions that determine deformation (conditions of securing). Therefore, we considered a particular but very characteristic and typical thermal problem, viz., the fraction of radiant light flux energy absorbed by the working surface is a source of thermal disturbances.

1. Let us consider a beam incident on a reflecting surface parallel to the principal optical axis OX which is brought to a focus by a mirror (see Fig. 1). The beam incident on the surface SOS' at a certain point A with the coordinates x_T and y_T will be reflected in the direction AB' and will intersect the principal optical axis at the point B'. After being reflected from point A^0, the extreme internal beam E^0 will intersect the main optical axis at the point B lying in the focal plane FF'. For a parabolic mirror in the absence of thermal distortions the points B and B' coincide.

Thermal deformations cause a change in the equation for the profile of the working surface of the mirror. This leads not only to a change in the focus, but also to the dependence of its value (more precisely of the segment OB) on the coordinate y (on the removal of the beam from the principal optical axis). The length of the segment BB' in the case where the point corresponds to the extreme external beam determines the value of the longitudinal spherical aberration Δ_1, whereas the corresponding segment Δ_2 (Fig. 1) determines the transverse spherical aberration [5].

The calculations of the thermooptical aberrations are based on an analytical formula which describes the coordinate of the mirror focus as a function of the distance between a ray of the axial beam and the principal optical axis [1]:

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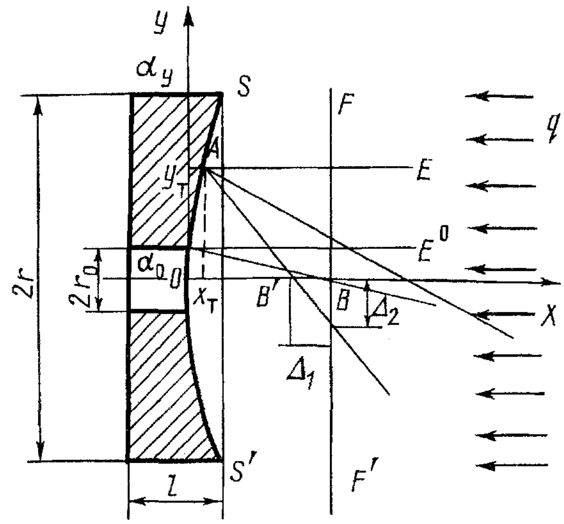

Fig. 1. Thermal and optical scheme of a mirror.

$$\bar{f} = \bar{x} + \bar{y}\frac{1 - \Phi^2}{2\Phi}; \quad \Phi = \frac{\partial \bar{x}}{d\bar{y}}; \quad \bar{f} = \frac{f}{R}; \quad \bar{x} = \frac{x}{R}; \quad \bar{y} = \frac{y}{R}, \tag{1}$$

In this case the dependence of $\bar{x}$ on $\bar{y}$, which corresponds to the equation for the profile of the working surface of mirrors with account for thermally induced distortion, can always be presented in the form of the functional relation (function) Q describing the thermally deformed profile [1-4]

$$\bar{x} = D_2 Q\left(\frac{\bar{y}}{D_1}\right); \quad D_k = 1 + \bar{U}_k; \quad k = 1, 2; \quad \bar{U}_1 = \frac{u}{y}; \quad \bar{U}_2 = \frac{w}{x}, \tag{2}$$

where the coefficients D_1 and D_2 take into account the radial u and axial w thermally induced displacements of the point on the working surface with the coordinates x and y. An easily comprehensible and compact result is obtained for one of the most typical versions of the working surface profile, i.e., parabolic, when [1]

$$Q\left(\frac{\bar{y}}{D_1}\right) = \frac{1}{2}\left(\frac{\bar{y}}{D_1}\right)^2 \text{ and } \bar{x} = \frac{D_2}{2D_1^2}\bar{y}^2; \tag{3}$$

the measure of the thermally induced distortion of the working surface profile can be described by the value of the displacement $\Delta\bar{x}$ with respect to the initial equation of the parabola $\bar{x}_0 = \bar{y}^2/2$

$$\Delta\bar{x} = \bar{x} - \bar{x}_0 = \left(\frac{D_2}{D_1^2} - 1\right)\frac{\bar{y}^2}{2} = \bar{W}\frac{\bar{y}^2}{2}; \quad \bar{W}|_{U_k \ll 1} = \bar{U}_2 - 2\bar{U}_1. \tag{4}$$

Substituting $\bar{x}$ and Φ into Eq. (1) with account for the increments $\bar{x} = \bar{x}_0 + \Delta\bar{x}$, $\Phi = \Phi_0 + \Delta\Phi$, where $\Phi_0 = \bar{y}$

$$\Delta\Phi = \bar{y}\left(\bar{W} + \frac{1}{2}\bar{y}\frac{d\bar{W}}{d\bar{y}}\right), \tag{5}$$

we easily obtain

$$\bar{f} = \frac{1}{2}\left[1 - \bar{W} - \frac{1}{2}\frac{d\bar{W}}{d\bar{y}}\bar{y}(1 + \bar{y}^2)\right]. \tag{6}$$

If we have information about the displacements characterized by one parameter $\bar{W}$, we can calculate the temperature displacement of the focus $\Delta\bar{f}^0$ and the value of the longitudinal spherical thermal aberration $\Delta\bar{f}$ determined from the relations

$$\Delta\bar{f}^0 = [\bar{f}(\overline{W}) - \bar{f}(\overline{W} = 0)]_{\bar{y}=\rho_0}, \tag{7}$$

$$\Delta\bar{f} = [\bar{f}(\bar{y} = \rho_0) - \bar{f}(\bar{y} = \rho)]. \tag{8}$$

2. Consider a particular thermal statement of the problem for a parabolic mirror. Hereafter, all of the calculations will be made for a mirror with a central hole. The general solution will yield as a specific case the solution for a solid mirror. Let us formulate the basic assumptions:

1) the heat flux q is constant within the change in the coordinate (see Fig. 1) $r_0 \le y \le r$ $(0 \le y \le r)$;
2) the thermal regime of the mirror is stationary;
3) the thermal properties of the material from which it is made are isotropic;
4) the mirror can be approximated by a disk upon satisfaction of the condition [1] $4\rho \le k$, $\rho = r/R$, $k = l/r$;
5) there is heat removal from the surfaces $x = 0$, $y = r_0$, $y = r$ (see Fig. 1) which is characterized by the heat transfer coefficients α_x, α_0, α_y, respectively, or by the numbers $\mathrm{Bi}_N = \alpha_N \xi/\lambda$; $N = 0, x, y$; $\xi = l, r$.

Let us consider a typical case where the surfaces x_0 and $y = r_0$ do not come in contact with heat sinks ($\mathrm{Bi}_x = \mathrm{Bi}_0 = 0$). This assumption leads to a one-dimensional-radial temperature field which is described, with an error not exceeding 2%, by an analytical solution of the form [2]

$$\vartheta(\bar{y}) = \vartheta_0\left[1 - B\left(\frac{\bar{y}}{\rho}\right)^2 + B_1 \ln\left(\frac{\bar{y}}{\rho}\right)\right], \quad B = \frac{\mathrm{Bi}_y}{2(1-\gamma^2) + \mathrm{Bi}_y},$$
$$B_1 = 2\gamma^2 B, \quad \gamma = \frac{r_0}{r}, \quad \vartheta = T - T_{\mathrm{am}}, \quad \vartheta_0 = \frac{\Delta\vartheta_0}{B}, \quad \Delta\vartheta_0 = \frac{qr^2}{4\lambda k}. \tag{9}$$

3. Consider the statement of the problem concerning the deformation of the profile of the working surface of a mirror in the case of free expansion and with a fixed boundary of the outer side surface. Let us restrict our discussion to an axisymmetric problem without twisting assuming the mechanical properties of the mirror material to be isotropic and the mirror to be a thin disk (the plane stressed state approximation $\sigma_x = 0$).

In this case the mathematical statement of the problem is reduced to the following form of the equilibrium equation and relationships between stresses and strains [6]:

$$\frac{\partial\sigma_y}{\partial y} + \frac{\sigma_y - \sigma_\theta}{y} = 0, \tag{10}$$

$$\varepsilon_y - \alpha\vartheta = \frac{1}{E}(\sigma_y - \nu\sigma_\theta), \quad \varepsilon_\theta - \alpha\vartheta = \frac{1}{E}(\sigma_\theta - \nu\sigma_y),$$
$$\varepsilon_x - \alpha\vartheta = -\frac{\nu}{E}(\sigma_y + \sigma_\theta), \tag{11}$$

where

$$\varepsilon_y = \frac{\partial u}{\partial y}, \quad \varepsilon_\theta = \frac{u}{y}, \quad \varepsilon_x = \frac{\partial w}{\partial x}.$$

The solution of systems (10) and (11) is known [6]:

$$u = (1+\nu)\alpha\frac{1}{y}\int_{r_0}^{y} y\vartheta\,dy + C_1 y + \frac{C_2}{y},$$

TABLE 1. Analytical Expressions for Intermediate Parameters

Parameter	Free boundary	Fixed boundary
$\overline{W}_j$	$\left[1 - B\left(\frac{\bar{y}}{\rho}\right)^2 + B_1 \ln\left(\frac{\bar{y}}{\rho}\right) - J_{1\rho}\frac{1}{1+\nu} - J_1\right](1+\nu)\,\delta h$	$\left[1 - B\left(\frac{\bar{y}}{\rho}\right)^2 + B_1 \ln\left(\frac{\bar{y}}{\rho}\right) + J_{2\rho}\left(\frac{\nu}{1-\nu}\varphi_0 + \varphi\right) - J_2\right](1+\nu)\,\delta h$
a_2	$-\frac{1+\nu}{2}B$	
a_{1j}	$b - \left(1 - \frac{B}{2}\right)$	$(1+\nu)\left\{\Omega\left(1 - \frac{B}{2}\right) + \gamma^2 B\,[1 - \Omega(1 + G_0)]\right\}$
a_{0j}	$(1+\nu)B\gamma^2 G_0$	$\gamma^2(1+\nu)\left\{1 + \Omega(1+\nu)\left[\left(1 - \frac{B}{2}\right) - B\gamma^2(1+G_0)\right] - \gamma^2 B\left(\frac{3}{2} - \ln\gamma^2\right)\right\}$
$\overline{F}_j$	$\left(1 - \frac{B}{2}\right) - B\gamma^2\,[1 + G_0(1 - m_1)] \approx 1 - B\left(\frac{1}{2} + \omega\right)$, when $\rho < 0.1$	$(1 - m_1)\Omega^*\left[B\left(\frac{1}{2} + \omega\right) - 1\right] + m_2 \approx -\Omega^*\left[1 - B\left(\frac{1}{2} + \omega\right)\right]$, when $\rho < 0.1$
$\Delta\overline{F}_j$	$a_2\,[\rho^2 + 2 - \gamma^2(m_3 + 2)] \approx 2a_2(1 - \gamma^2)$, when $\rho < 0.1$	

$$\sigma_y = -\alpha E \frac{1}{y^2}\int_{r_0}^{y} y\vartheta\,dy + \frac{E}{1-\nu^2}\left[C_1(1+\nu) - C_2(1-\nu)\frac{1}{y^2}\right],$$

$$\sigma_\theta = \alpha E \frac{1}{y^2}\int_{r_0}^{y} y\vartheta\,dy - \alpha E\vartheta + \frac{E}{1-\nu^2}\left[C_1(1+\nu) + C_2(1-\nu)\frac{1}{y^2}\right]. \tag{12}$$

Substituting expressions for stresses (12) into the last equation of system (11), we obtain a formula for determining the axial displacements ϖ

$$w = \left[\alpha\vartheta(1+\nu) - \frac{2\nu}{1-\nu}C_1\right]x. \tag{13}$$

To find the constants C_1 and C_2, we shall avail ourselves of the following boundary conditions:

1. The case of free expansion $(\sigma_y(y = r) = 0)$. Hence

$$C_1^{\mathrm{fr}} = \frac{\alpha}{r^2 - r_0^2}(1-\nu)\int_{r_0}^{r} y\vartheta\,dy.$$

2. The case of a fixed boundary $(u(y = r = 0)$:

$$C_1^{\mathrm{f}} = \frac{(1-\nu^2)\alpha}{(1-\nu)r^2 + (1+\nu)r_0^2}\int_{r_0}^{r} y\vartheta\,dy,$$

$$C_1^{\mathrm{f}} = -C_1^{\mathrm{fr}}(1+\nu)\,\Omega\,,\quad \Omega = \frac{1-\gamma^2}{(1-\nu)+(1+\nu)\gamma^2},$$

C_2 is sought proceeding from the condition of the absence of stresses on the hole boundary: $\sigma_y(y = r_0) = 0$; $C_2^{\mathrm{fr,f}} = C_1^{\mathrm{fr,f}}\, r_0^2\,(1+\nu)/(1-\nu)$. Using relations (2) and (4) for a temperature field of form (9), we obtain analytical expressions for $\overline{W}_j$, where the subscript $j = 1, 2$ corresponds to free expansion 1, and fixed boundary 2 for all the cases considered. The values of $\overline{W}_j$ are given in Table 1, where $\delta h = \alpha\vartheta_0$. In Table 1 the following designations are adopted:

$$J_1 = 1 - \frac{B}{2}\left[\gamma^2\left(\frac{\bar y}{\rho}\right)^{-2} + \left(\frac{\bar y}{\rho}\right)^{2}\right] - \frac{B_1}{2}\left[1 - \ln\left(\frac{\bar y}{\rho}\right)^{2} + \frac{\gamma^2\ln\gamma^2}{1-\gamma^2}\left(\frac{\bar y}{\rho}\right)^{-2}\right],$$

$$J_2 = 1 - \gamma^2\left(\frac{\bar y}{\rho}\right)^{-2} - \frac{B}{2}\left[\left(\frac{\bar y}{\rho}\right)^{2} - \gamma^4\left(\frac{\bar y}{\rho}\right)^{-2}\right] -$$

$$-\frac{B_1}{2}\left[1 - \ln\left(\frac{\bar y}{\rho}\right)^{2} + \gamma^2\left(\frac{\bar y}{\rho}\right)^{-2}(\ln\gamma^2 - 1)\right],$$

$$J_{1\rho} = J_1(\bar y = \rho)\quad J_{2\rho} = J_2(\bar y = \rho) = (1-\gamma^2)\,J_{1\rho},$$

$$\varphi = \frac{1 + \tilde\nu\gamma^2\left(\frac{\bar y}{\rho}\right)^{-2}}{1+\tilde\nu\gamma^2},\quad \tilde\nu = \frac{1+\nu}{1-\nu},\quad \varphi_0 = \frac{1}{1+\tilde\nu\gamma^2}.$$

Expanding all the components involved in the expressions in Table 1, we obtain

$$\overline{W}_j = \delta h \sum_{i=0}^{2} a_{ij}\left(\frac{\bar y}{\rho}\right)^{2(i-1)}. \tag{14}$$

The values of the coefficients a_{ij} are also included into Table 1 where the following designations are adopted:

$$b = B\gamma^2\,[(1+\nu) + (1+G_0)],\quad G_0 = \frac{1}{2} + G,\quad G = \frac{\gamma^2\ln\gamma^2}{1-\gamma^2}.$$

Substituting Eq. (14) into Eq. (4), we obtain a formula for the measure of the distortion of the profile $\Delta\bar x_j$:

$$\Delta\bar x_j = \frac{\delta h}{2}\rho^2\sum_{i=0}^{2} a_{ij}\left(\frac{\bar y}{\rho}\right)^{2i}. \tag{15}$$

4. Substituting Eq. (14) into Eq. (6) with account for Eqs. (7) and (8), we obtain the resultant formulas for $\bar f_j$, $\Delta\bar f_j^0$, and $\Delta\bar f_j$

$$\bar f_j = -\frac{1}{2}\left\{\delta h\sum_{i=0}^{2}\left\{a_{ij}\left(\frac{\bar y}{\rho}\right)^{2(i-1)}[i(1+\bar y^2) - \bar y^2]\right\} - 1\right\},$$

$$\Delta\bar f_j^0 = -\frac{1}{2}\delta h\sum_{i=0}^{2} a_{ij}\gamma^{2(i-1)}\,[i(1+\rho^2\gamma^2) - \rho^2\gamma^2] =$$

$$= \frac{\delta h}{2} [\rho^2 a_{0j} - a_2 \gamma^2 (\rho^2 \gamma^2 + 2) - a_{1j}]_{\bar{y}=\rho_0} = \frac{\delta h}{2} \bar{F}_j (\bar{y} = \rho_0) , \tag{16}$$

$$\begin{aligned} \Delta \bar{f}_j &= - \frac{1}{2} \delta h \sum_{i=0}^{2} a_{ij} \left\{ \rho^2 (\gamma^{2i} - 1) - i [\gamma^{2(i-1)} + \rho^2 (\gamma^{2i} - 1) - 1] \right\} = \\ &= \frac{\delta h}{2} \left\{ 2a_2 (1 - \gamma^2) \left[1 + \frac{\rho^2}{2} (1 + \gamma^2) \right] \right\} = \frac{\delta h}{2} \Delta \bar{F}_j . \end{aligned} \tag{17}$$

The expressions for $\bar{F}_j$ and $\Delta \bar{F}_j$ are given in Table 1, where the following designations are used:

$$m_1 = \rho^2 \gamma^2 (1 + \nu) , \quad m_2 = (1 + \nu) \rho^2 \gamma^2 \left\{ 1 - B \left[\frac{1}{2} (3 + \gamma^2) - \ln \gamma^2 \right] \right\} ,$$

$$m_3 = \rho^2 \gamma^2 , \quad \Omega^* = (1 + \nu) \Omega , \quad \omega = \gamma^2 (1 + G_0) .$$

When $\rho < 0.1$, the expressions in Table 1 can be simplified by neglecting the values $m_1 - m_3$ due to their smallness except for the case $\gamma \geq 0.7$ (in the formula for $\bar{F}_1$) and $\gamma \geq 0.3$ (for $\bar{F}_2$) when the error exceeds 1% and in some cases 10%. Table 1 contains the formulas in their simplified form. As a result, for long-focus mirrors the finite expressions can be presented in a very simple form

$$\Delta \bar{x}_j = \frac{\delta h}{2} \rho^2 \left[a_0 + a_1 \left(\frac{\bar{y}}{\rho} \right)^2 + a_2 \left(\frac{\bar{y}}{\rho} \right)^4 \right] , \tag{18}$$

$$\Delta \bar{f}_1^0 = \frac{\delta h}{2} \left[1 - B \left(\frac{1}{2} + \omega \right) \right] , \tag{19}$$

$$\Delta \bar{f}_2^0 = - \Omega^* \Delta \bar{f}_1^0 , \tag{20}$$

$$\Delta \bar{f}_{1,2}^0 = - \frac{\delta h}{2} (1 + \nu) B (1 - \gamma^2) . \tag{21}$$

The transverse spherical thermal aberration is expressed in terms of the longitudinal one [1]

$$\Delta \bar{f}_1^{\mathrm{tr}} = 2\rho \, \Delta \bar{f}_{1,2}^0 . \tag{22}$$

For convenience of analysis of the thermal distortions in the working surface profile of the mirror, Fig. 2a presents the relations for the coefficients $a_{0j}(\gamma)$ and $a_{1j}(\gamma)$ in Eq. (18) for determining the displacements of the focus $\Delta \bar{f}_j^0$ (Fig. 2b).

The results obtained make it possible to draw the following conclusions.

1. Since the function of the displacements $\Delta \bar{x}$ is incorporated in the general algorithm for determining the thermal aberrations as the principal determining parameter, with its derivative used as the second parameter, the algorithm is general, whereas the specific results are determined by the form of the function of displacements which depends on the initial thermal problem and the conditions of fixing the mirror. The present work considers in analytical form for the first time the effect of the limitation of thermal expansion on the development of thermal deformations and thermal aberrations.

2. With the considered (and other) types of thermal effects leading to a radial temperature profile of the form given by Eq. (9), the thermal deformations distort the parabolic profile up to the 4th degree of asphericity.

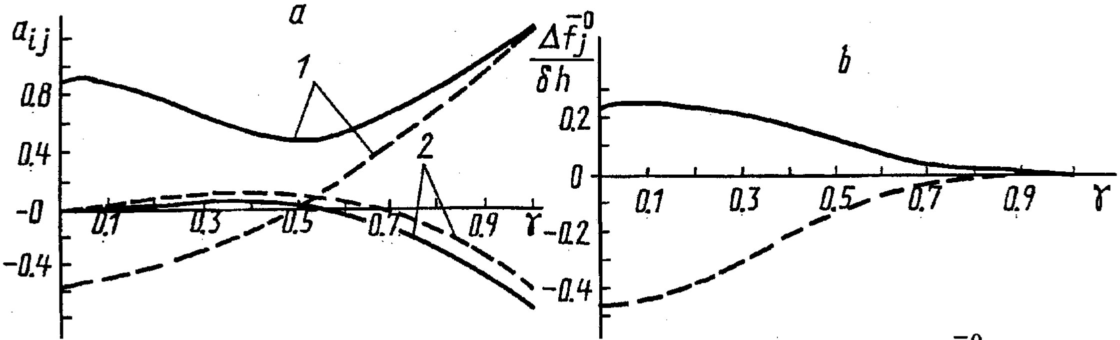

Fig. 2. The coefficients a_{ij}: 1) a_{1j}; 2) a_{0j}; (a) and relative values of $\Delta\bar{f}_j^0/\delta h$ (b) vs the coefficients of central screening. Solid lines, free expansion (j = 1), dashed line, fixed boundary (j = 2). In calculations $\nu = 0.3$, $B = 1$.

3. Within the scope of the thermal problem considered, the temperature displacement of the focus with different means of fixing (free expansion and fixed boundary of the outer surface) differs in sign and value. For a solid mirror at $\nu = 0.3$ this difference turns out to be twofold.

4. For the extreme cases of fixing considered, the longitudinal spherical thermal aberration is identical and has a negative sign, i.e., the beams reflected by the peripheral portions of the mirror intersect the principal optical axis at the points located at a larger distance from the mirror than the paraxial focus. In the case of uniform heating the aberration is absent.

5. The resulting analytical solutions (18)-(22) are extremely convenient for practical estimates.

As an example, we can evaluate the threshold values of temperature deformations. Prescribing the aberration limitation and proceeding from the Rayleigh number $|\Delta f| \leq \Lambda/4$ we can easily see from Eq. (21) that $|\delta h^{\lim}| \leq \Lambda/2(1+\nu)R$ for $B = 1$ and $\gamma = 0$. Taking $\Lambda = 0.7\cdot10^{-6}$ m, $R = 1$ m, $\nu = 0.3$ as the initial values, we obtain $|\delta h^{\lim}| \leq 2.7\cdot10^{-7}$. At $\alpha = 10^{-5}$ K^{-1} this corresponds to the ultimate admissible temperature drop $|\vartheta_0|_{B=1} = \Delta\vartheta \lesssim 0.03$ K. The heat flux corresponding to such a temperature drop for a mirror with parameters $r = 10^{-1}$ m, $\rho = 0.1$, k = 0.4, $\lambda = 10$ W/mK amounts to ≈ 5 W/M^2, which roughly corresponds to the absorption of 0.3% of the power of the solar light flux by the mirror surface. The displacement of the focus in the example considered will come to $\Delta f_1^0 = R\delta h^{\lim}/4 = +0.07$ μm in the case of free expansion and to $\Delta f_2^0 = -(1+\nu)/(1-\nu)\Delta f_1^0 \approx -0.14$ μm for a fixed outer boundary of the mirror. Moreover, in both cases $\Delta f_{1,2} = -0.175$ μm.

NOTATION

T, temperature, K; T_{am}, ambient temperature, K; ϑ, superheating above the ambient temperature, K; λ, thermal conductivity coefficient of the mirror material, W/(m·K); x, y, coordinates; r, radius of mirror, m; r_0, radius of hole in mirror, m; l, mirror thickness, m; q, heat flux density, W/m^2; α_0, α_x, α_y, heat transfer coefficients on corresponding surfaces, W/(m^2·K); σ_y, σ_θ, σ_x, normal components of stresses in cylindrical coordinates; ε_y, ε_θ, ε_x, deformations corresponding to these components; E, Young modulus; ν, Poisson coefficient; u, w, components of radial and axial displacement; α, coefficient of thermal expansion, K^{-1}; f, focal length of mirror, m; R, paraxial radius of mirror, m; L, wave length, mm.

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